48th Annual Dick Schaff Math Superbowl

2023 Level 6 Exam – Calculus Blitz

Directions:
1. Select the most correct answer for each question and mark it on your Scantron.
2. NO CALCULATORS OR ELECTRONIC DEVICES MAY BE USED.
3. Note that N.O.T. means "none of these."

1. The value of $\lim_{x\to 0}(\cot(5x)\tan(3x))$ is

a) 0 b) $\frac{5}{3}$ c) 1 d) $\frac{3}{5}$ e) N.O.T.

2. The acceleration of a particle can be modeled by $a(t) = 3t^2 - 6t$ on $[0,10]$. If the particle is initially at rest and has an initial position of 0 find the position of the particle at $t = 10$.

a) 1500 b) 1000 c) 6 d) 0 e) N.O.T.

3. Find the tangent line to the equation $e^{2y} - e^{-2y} = 2\sin(2x)$ at $(x, y) = (\pi, 0)$.

a) $y = -\pi - x$ b) $y = -\pi + x$ c) $y = \pi + x$ d) $y = \pi - x$ e) N.O.T.

4. Let $f(x) = \cos(x) - \sin(x)$ on $(0, 2\pi)$. The sum of the x-value(s) where $f(x)$ has local extrema is

a) 3π b) π c) 2π d) 0 e) N.O.T.

5. Let $f'(x) = e^x(5x^2 - 16x + 17)$. Then f is always concave down on which of the following intervals?

a) $(\frac{1}{5}, 1)$ b) $(-\frac{1}{5}, 1)$ c) $(1, \infty)$ d) $(0, 1)$ e) N.O.T.

6. The value of $\lim_{x\to 0}\frac{\cos(3x)-1}{x^2}$ is:

a) $\frac{9}{2}$ b) 1 c) $-\frac{3}{2}$ d) 0 e) N.O.T.

7. The sum of the critical values of $f(x) = e^x(x-1)^2$ is:

a) 0 b) 2 c) e d) 1 e) N.O.T.

8. Compute $\lim_{x\to-\infty}\frac{4x^2-5x}{4x^2-3x}$

a) $-\frac{1}{2}$ b) -2 c) 1 d) $\frac{5}{3}$ e) N.O.T.

9. Find the value of c that satisfies the conditions of the Mean Value Theorem on the interval $[1,8]$ for the function $f(x) = \log_{10}(x)$.

a) $\frac{\ln 8}{7}$ b) $\frac{7\ln 10}{3\ln 2}$ c) $\frac{3}{7}\log_{10} 2$ d) $\frac{7}{3\ln 2}$ e) N.O.T.

10. Find the area of the region bounded by the curves $y = 2 - x^2$ and $y = -x - 4$.

a) $\frac{125}{6}$ b) $\frac{81}{6}$ c) $\frac{5}{2}$ d) 9 e) N.O.T.

11. Evaluate $\lim_{h\to 0} \frac{(9+h)^{-1/2} - (9)^{-1/2}}{h}$

a) $\frac{-27}{2}$ b) -3 c) $-\frac{1}{54}$ d) $-\frac{1}{27}$ e) N.O.T.

12. If $f(x) = \int_1^{\csc x} (e^t + t^2)\, dt$, then $f'\left(\frac{\pi}{2}\right) =$

a) 0 b) $e + 1$ c) $-e - 1$ d) 1 e) N.O.T.

13. Let $f(x) = \begin{cases} x + 16 & \text{if} \quad x \le -2 \\ ax^2 + b - 4 & \text{if} \quad -2 < x \le 1 \\ 3bx - ax^2 & \text{if} \quad x > 1 \end{cases}$

Using the value of the constants a and b that make the function continuous everywhere, find $a + b$.

a) 0 b) -1 c) 1 d) 2 e) N.O.T.

14. The derivative of $f(x) = \cot^{-1}\left(\frac{2x}{5}\right)$ is

a) $\frac{2}{\sqrt{25-4x^2}}$ b) $-\frac{2}{\sqrt{25-4x^2}}$ c) $-\frac{10}{25+4x^2}$ d) $\frac{10}{25+4x^2}$ e) N.O.T.

15. Find the slope of the tangent line to the curve $\cos(x + y) = x - 2y$ at the point $\left(\pi, \frac{\pi}{2}\right)$.

a) $\frac{1}{2}$ b) -1 c) 0 d) $-\frac{1}{2}$ e) N.O.T.

16. If $f(x) = \begin{cases} \ln x & if \quad x \le 1 \\ \sin\left(\frac{\pi}{x-1}\right) & if \quad x > 1 \end{cases}$ then $\lim_{x\to 1^-} f(x) + \lim_{x\to 2^+} f(x)$ is

a) $1 + e$ b) -1 c) $e - \frac{1}{2}$ d) 1 e) N.O.T.

17. Evaluate $\int_{-3}^{3} [5 - |x - 2|]\, dx$

a) $\frac{25}{2}$ b) $\frac{9}{2}$ c) $\frac{33}{2}$ d) 15 e) N.O.T.

18. If $f(x) = \ln((bx)^a)$ where a, b are both positive constants, what is the value of $f'(1)$?

a) b b) a c) $-a/b$ d) a/b e) N.O.T.

19. If $f'(x) = \tan(3x+1)$ and $f\left(-\frac{1}{3}\right) = 0$ then $f(x) =$

a) $\frac{1}{3}\ln|\cos(3x+1)|$ b) $\frac{1}{3}\ln|\sec(3x+1)|$ c) $\frac{1}{3}\ln(\cos(3x+1))$ d) $\frac{1}{3}\ln(\sec(3x+1))$ e) N.O.T.

20. If $f(x) = \int_1^{\ln x} e^{2t}\,dt$ then $f'(2) =$

a) e^2 b) e^4 c) $e^{2\ln 2}$ d) e e) N.O.T.

21. Compute $\lim_{x\to 1^+} (\ln x)^x$

a) $-\infty$ b) 0 c) ∞ d) 1 e) N.O.T.

22. Find the value of $\int_0^1 x\sqrt{9-5x^2}\,dx$

a) $\frac{1}{15}$ b) $\frac{14}{15}$ c) $-\frac{36}{15}$ d) $\frac{19}{15}$ e) N.O.T.

23. Using the table below, find the difference between approximating $\int_{10}^{50} f(x)dx$ using the rectangular method with right endpoints and the rectangular method with left endpoints. Use rectangles of width 10.

x	0	10	20	30	40	50	60
$f(x)$	7	2	5	-6	-4	8	3

a) 40 b) 0 c) -40 d) 60 e) N.O.T.

24. The value of $\lim_{x\to 0^+} x^x$ is

a) 1 b) ∞ c) $\frac{1}{e}$ d) 0 e) N.O.T.

25. A baseball diamond is a square with side 90 ft. A batter hits the ball and runs toward first base with a speed of 26 ft/s. At what rate is his distance from second base decreasing when he is halfway to first base?

a) $\frac{12}{\sqrt{5}}$ ft/s b) $\frac{90}{\sqrt{5}}$ ft/s c) $\frac{26}{\sqrt{5}}$ ft/s d) $\frac{3}{\sqrt{5}}$ ft/s e) N.O.T.

26. What is the value of $\lim_{x\to 3} f'(x)$ if $f(x) = -4|x-3|$

a) 4 b) -4 c) 1 d) 0 e) N.O.T.

27. The average value of $f(x) = \sin(2x)$ on $\left[-\pi, \frac{3\pi}{2}\right]$ is

a) $\frac{1}{2}$ b) $\frac{5\pi}{2}$ c) $\frac{2}{5\pi}$ d) 0 e) N.O.T.

28. Evaluate $\int_{\pi/6}^{\pi/3} \tan(\theta)\sec^2(\theta)\,d\theta$.

a) $\frac{5}{3}$ b) $\frac{4}{3}$ c) $\frac{8}{3}$ d) 3 e) N.O.T.

29. Find $f^{(914)}(x)$ if $f(x) = \cos(x)$.

a) $\sin(x)$ b) $\cos(x)$ c) $-\sin(x)$ d) $-\cos(x)$ e) N.O.T.

30. Suppose that $\int_7^{10} g(x)dx = 3$, $\int_5^7 g(x)dx = 8$, and $\int_{15}^{10} g(x)dx = -1$. Find $\int_5^{15} g(x)dx$.

a) -4 b) -24 c) 10 d) 12 e) N.O.T.

31. Given $x^2 + y^2 = \sin(xy) + 1$, then the slope of the tangent line at $(x, y) = (1,0)$ is

a) 2 b) -1 c) $-\frac{1}{2}$ d) 0 e) N.O.T.

32. The sum of the *y*-coordinates of the horizontal asymptote(s) of $f(x) = \arctan(2x) + 1$ is

a) $\frac{\pi}{2}$ b) 2 c) 1 d) 0 e) N.O.T.

33. A hemisphere with a 5-meter radius is to be coated in an even layer of paint 2mm thick. Use differentials to estimate how much paint is needed (in m^3). Assume the bottom of the hemisphere is on the ground and does not need to be painted.

a) $\frac{\pi}{20}$ b) $\frac{\pi}{10}$ c) 100π d) 10π e) N.O.T.

34. Evaluate $\int \frac{2x}{1+16x^4}dx$.

a) $\frac{\tan^{-1}(4x^2)}{4} + C$ b) $\frac{\tan^{-1}(4x^4)}{4} + C$ c) $\frac{\tan^{-1}(16x^4)}{2} + C$ d) $\frac{\tan^{-1}(4x^4)}{2} + C$ e) N.O.T.

35. A piece of wire 12 meters long is bent into the shape of a rectangle. Find the maximum area that can be enclosed by such a rectangle.

a) 6 m^2 b) 3 m^2 c) 9 m^2 d) $6\sqrt{2}$ m^2 e) N.O.T.

36. Which of the following points on the ellipse $x^2 + 4y^2 = 16$ is furthest from the point (0, 5)?

a) $\left(-\frac{\sqrt{44}}{3}, -\frac{5}{3}\right)$ b) $\left(\frac{\sqrt{44}}{3}, \frac{5}{3}\right)$ c) (0,2) d) $(0, -2)$ e) N.O.T.

37. Evaluate $\int \tan^2(x)\,dx$

a) $\sec^2(x) + C$ b) $\tan(x) + x + C$ c) $\tan(x) - x + C$ d) $\sec^2(x) + x + C$ e) N.O.T.

38. If $f(x) = 3^{\ln(\cos(x^2+2x))}$ then what is the value of $f'(0)$?

a) -3 b) 0 c) $6\ln 3$ d) $3\ln 3$ e) N.O.T.

39. Let $f(x) = \frac{x^2}{x^2+6}$. What is the difference between the two x-values of the inflection points of f?

a) 12 b) -6 c) $\sqrt{2}$ d) $2\sqrt{2}$ e) N.O.T.

40. At which of the following x-values is the function $f(x) = \frac{x^2+3x+2}{24x^2+2x-15}$ not continuous?

a) -2 b) $-\frac{5}{6}$ c) -1 d) $-\frac{3}{4}$ e) N.O.T.